Subject : Mathematics

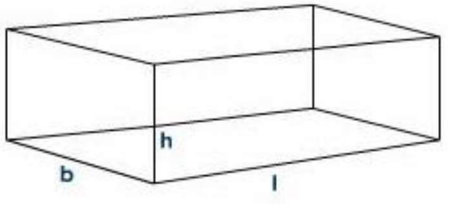
Topic: Surface Area and Volume

General Instructions

1. The solutions for Exercises of NCERT Chapter 13 – Surface Area and Volumes are to be done in the register.
2. The solutions for the assignment given at the end are to be done in separate
   A4 sheets.

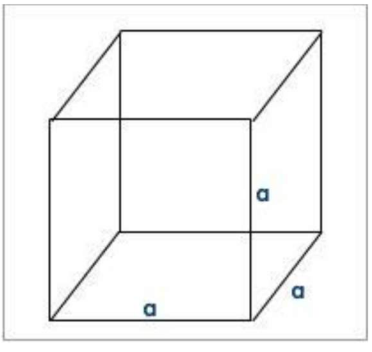
Surface Areas and Volumes

Cuboid



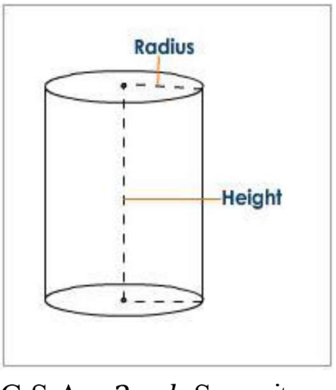
Total surface area = 2 (lb + bh + hl) Sq. units
Lateral surface area = Area of 4 walls = 2h (l + b) Sq. units
Volume = lbh Cu. units

Cube



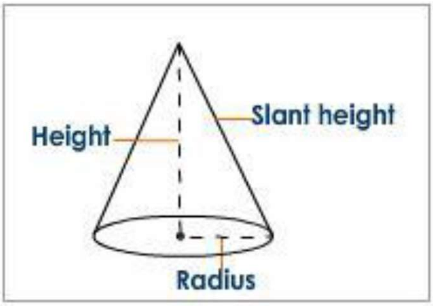
Let each side be 'a' units
Total surface area = $6a^2$ Sq. units
Lateral surface area = $4a^2$ Sq. units
Volume = a^3 Cu. units

Cylinder



C.S.A = $2\pi rh$ Sq. units
T.S.A = $2\pi r(h + 1)$ Sq. units
Volume = $\pi r^2 h$ Cu. units

Cone

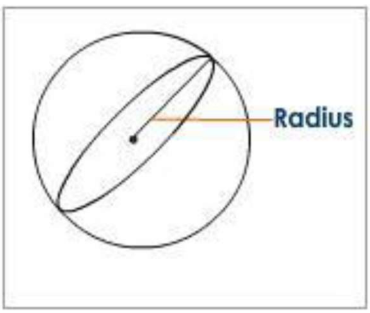


C.S.A = $\pi r l$ sq. units

T.S.A = $\pi r\ (l + r)$sq. units

Volume $= \frac{1}{3}\ \pi r^2 h$ cu. units

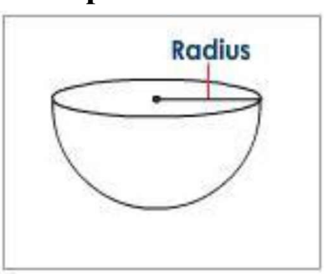
Sphere



Area = $4\pi r^2$Sq. units

Volume $= \frac{4}{3}\ \pi r^3$ Cu. units

Hemisphere

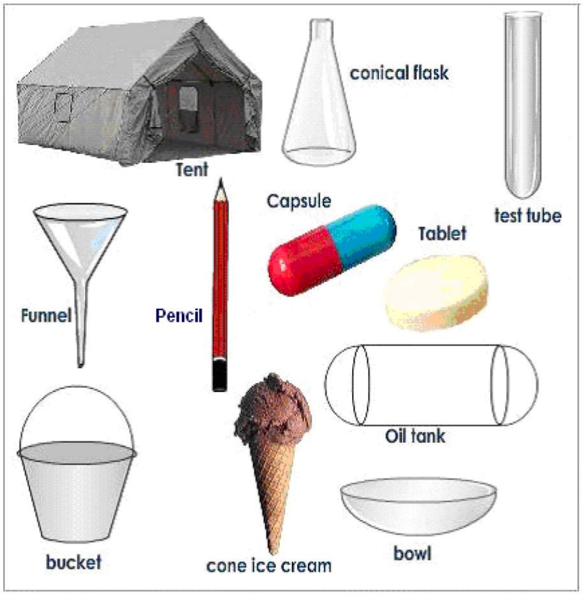


C.S.A = $2\pi r^2$ Sq. units

T.S.A = $3\pi r^2$Sq. units

Volume $= \frac{2}{3}\ \pi r^2$ Cu. units

Surface Area of a Combination of Solids



Total surface area of such a combined solid is found by adding the curved surface areas of the individual parts.

Volume of a Combination of Solids

Volume of the new solid formed by the combination of solids is the sum of the volumes of the individual solids.

Conversion of solid from one Shape to another

When one solid is converted into other solid, then their volumes are the same.

EXERCISE NO:13.1

Question 1:

2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboids.

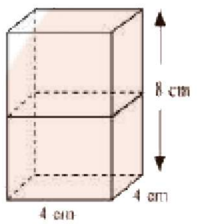
Solution 1:

Given that,

Volume of cubes = 64 cm^3

$(\text{Edge})^3 = 64$

Edge = 4 cm



If cubes are joined end to end, the dimensions of the resulting cuboid will be 4 cm, 4cm, 8 cm.

$\therefore$ Surface area of cuboids $= 2(\text{lb} + \text{bh} + \text{lh})$

$$= 2(4\times 4 + 4\times 8 + 4\times 8)$$

$$= 2(16 + 32 + 32)$$

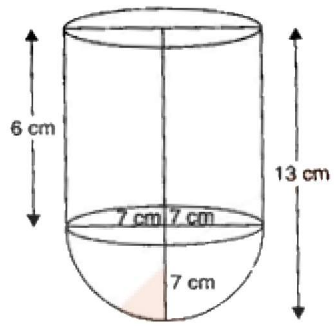
$$= 2(16 + 64)$$

$$= 2\times 80 = 160 \text{ cm}^2$$

Question 2:

A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel. $\left[\text{Use } \pi = \dfrac{22}{7}\right]$

Solution 2:



It can be observed that radius (r) of the cylindrical part and the hemispherical part is the same (i.e., 7 cm).Height of hemispherical part = Radius = 7 cm
Height of cylindrical part (h) = 13 − 7 = 6 cm
Inner surface area of the vessel = CSA of cylindrical part + CSA of hemispherical part

$$= 2\pi rh + 2\pi r^2$$

Inner surface area of vessel = $2 \times \frac{22}{7} \times 7 \times 6 + 2 \times \frac{22}{7} \times 7 \times 7$

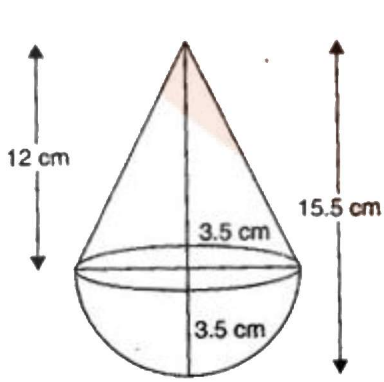
$$= 44(6+7) = 44 \times 13$$

$$= 572\ \text{cm}^2$$

Question 3:

A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Solution 3:



It can be observed that the radius of the conical part and the hemispherical part is same (i.e., 3.5 cm).

Height of hemispherical part = Radius (r) = 3.5 = $\frac{7}{2}$ cm

Height of conical part (h) = 15.5 −3.5 = 12 cm

Slant height (l) of conical part = $\sqrt{r^2 + h^2}$

$$= \sqrt{\left(\frac{7}{2}\right)^2 + (12)^2} = \sqrt{\frac{49}{4} + 144} = \sqrt{\frac{49 + 576}{4}}$$

$$= \sqrt{\frac{625}{4}} = \frac{25}{2}$$

Total surface area of toy = CSA of conical part + CSA of hemispherical part

$$= \pi rl + 2\pi r^2$$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{25}{2} + 2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

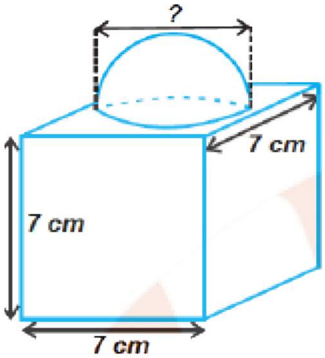
$$= 137.5 + 77 = 214.5cm^2$$

Question 4:

A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

$\left[\text{Use } \pi = \frac{22}{7}\right]$

Solution 4:



From the figure, it can be observed that the greatest diameter possible for such hemisphere is equal to the cube's edge, i.e., 7cm.

Radius (r) of hemispherical part $= \frac{7}{2} = 3.5\text{cm}$

Total surface area of solid = Surface area of cubical part + CSA of hemispherical part
− Area of base of hemispherical part
$= 6\,(\text{Edge})^2 + 2\pi r^2 - \pi r^2 = 6\,(\text{Edge})^2 + \pi r^2$

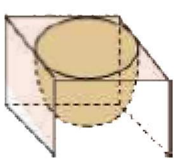
Total surface area of solid $= 6(7)^2 + \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$= 294 + 38.5 = 332.5\ \text{cm}^2$

Question 5:

A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Solution 5:



Diameter of hemisphere = Edge of cube = l

Radius of hemisphere $\frac{l}{2}$

Total surface area of solid = Surface area of cubical part + CSA of hemispherical part
− Area of base of hemispherical part
$= 6\,(\text{Edge})^2 + 2\pi r^2 - \pi r^2 = 6\,(\text{Edge})^2 + \pi r^2$

Total surface area of solid $= 6l^2 + \pi \times \left(\frac{l}{2}\right)^2$

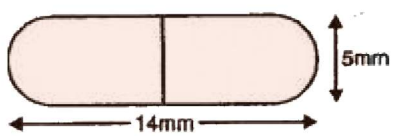
$$= 6l^2 + \frac{\pi l^2}{4}$$

$$= \frac{1}{4}(24 + \pi) l^2 \text{ unit}^2$$

Question 6:

A medicine capsule is in the shape of cylinder with two hemispheres stuck to each of its ends (see the given figure). The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.

$\left[\text{Use } \pi = \frac{22}{7}\right]$



Solution 6:

It can be observed that

Radius (r) of cylindrical part = Radius (r) of hemispherical part

$$= \frac{\text{Diameter of the capsule}}{2} = \frac{5}{2}$$

Length of cylindrical part (h) = Length of the entire capsule − 2 × r

$$= 14 - 5 = 9 \text{ mm}$$

Surface area of capsule = 2CSA of hemispherical part

+ CSA of cylindrical part

$= 2 \times 2\pi r^2 + 2\pi rh$

$= 4\pi \left(\frac{5}{2}\right)^2 + 2\pi \left(\frac{5}{2}\right)(9)$

$= 25\pi + 45\pi$

$= 70\pi$

$= 70 \times \frac{22}{7}$

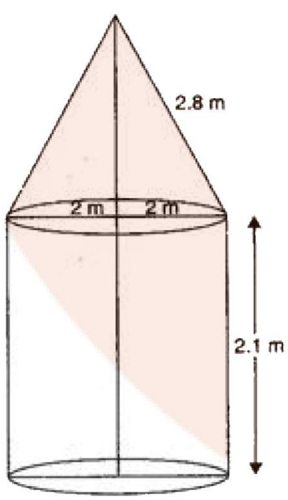
$= 220 \text{ mm}^2$

Question 7:

A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs 500 per m^2. (Note that the base of the tent will not be covered with canvas.)

$\left[\text{Use } \pi = \frac{22}{7}\right]$

Solution 7:



Given that,

Height (h) of the cylindrical part = 2.1 m

Diameter of the cylindrical part = 4 m

Radius of the cylindrical part = 2 m

Slant height (l) of conical part = 2.8 m

Area of canvas used = CSA of conical part + CSA of cylindrical part

$= \pi rl + 2\pi rh$

$= \pi \times 2 \times 2.8 + 2\pi \times 2 \times 2.1$

$= 2\pi[2.8 + 2 \times 2.1]$

$= 2\pi[2.8 + 4.2] = 2 \times \frac{22}{7} \times 7$

$= 44m^2$

Cost of 1 m^2 canvas = Rs 500

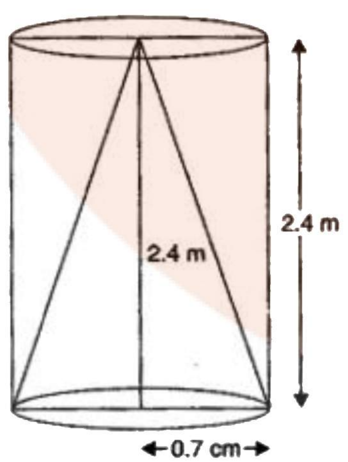
Cost of 44 m^2 canvas = 44 × 500 = 22000

Therefore, it will cost Rs 22000 for making such a tent.

Question 8:

From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Solution 8:



Given that,

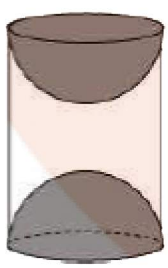
Height (h) of the conical part = Height (h) of the cylindrical part = 2.4 cm

Diameter of the cylindrical part = 1.4 cm
Therefore, radius (r) of the cylindrical part = 0.7 cm
Slant height (l) of conical part = $\sqrt{r^2 + h^2}$
$= \sqrt{(0.7)^2 + (2.4)^2} = \sqrt{0.49 + 5.76}$
$= \sqrt{6.25} = 2.5$

Total surface area of the remaining solid will be
= CSA of cylindrical part + CSA of conical part + Area of cylindrical base
$= 2\pi rh + \pi rl + \pi r^2$
$= 2 \times \frac{22}{7} \times 0.7 \times 2.4 + \frac{22}{7} \times 0.7 \times 2.5 + \frac{22}{7} \times 0.7 \times 0.7$
$= 4.4 \times 2.4 \times + 2.2 \times 2.5 + 2.2 \times 0.7$
$= 10.56 + 5.50 + 1.54 = 17.60 \text{ cm}^2$
The total surface area of the remaining solid to the nearest cm^2 is 18 cm^2

Question 9:
A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in given figure. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article.
$\left[\text{Use } \pi = \frac{22}{7}\right]$

Solution 9:



Given that,
Radius (r) of cylindrical part = Radius (r) of hemispherical part = 3.5 cm
Height of cylindrical part (h) = 10 cm
Surface area of article = CSA of cylindrical part + 2 × CSA of hemispherical part

$= 2\pi \text{rh} + 2 \times \pi \text{r}^2$

$= 2\pi \times 3.5 \times 10 + 2 \times 2\pi \times 3.5 \times 3.5$

$= 70\pi + 49\pi$

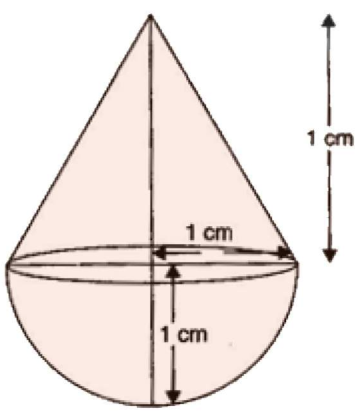
$= 119\pi$

$= 17 \times 22 = 374 \text{ cm}^2$

EXERCISE NO:13.2

Question 1:

A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.



Solution 1:

Height (h) of conical part = Radius(r) of conical part = 1 cm
Radius(r) of hemispherical part = Radius of conical part (r) = 1 cm
Volume of solid = Volume of conical part + Volume of hemispherical part

$= \frac{1}{3}\pi \text{r}^2 \text{h} + \frac{2}{3}\pi \text{r}^3$

$= \frac{1}{3}\pi(1)^2(1) + \frac{2\pi}{3}\pi(1) = \frac{2\pi}{3} + \frac{}{3} = \pi \text{ cm}^2$

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

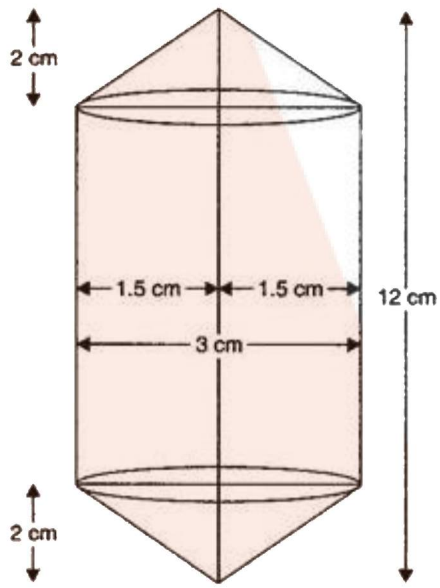
$$= \frac{1}{3}\pi(1)^2 1 + \frac{2}{3}\pi(1)^3$$

$$= \frac{3\pi}{3} = \pi cm^2$$

Question 2:

Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminum sheet. The diameter of the model is 3 cm and its length is 12 cm. if each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.) $\left[\text{Use } \pi = \frac{22}{7}\right]$

Solution 2:



From the figure, it can be observed that

Height (h_1) of each conical part = 2 cm

Height (h_2) of cylindrical part = 12 − 2 × Height of conical part

= 12 − 2 ×2 = 8 cm

Radius (r) of cylindrical part = Radius of conical part = $\frac{3}{2}$ cm

Volume of air present in the model

= Volume of cylinder + 2 × Volume of cones

$$= \pi r^2 h_2 + 2 \times \frac{1}{3} \pi r^2 h_1$$

$$=\pi\left(\frac{3}{2}\right)^2(8)+2\times\frac{1}{3}\pi\left(\frac{3}{2}\right)^2\ (2)$$

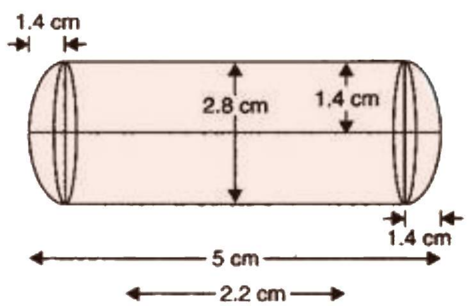
$$=\pi\times\frac{9}{4}\times 8+\frac{2}{3}\pi\times\frac{9}{4}\times 2$$

$$=18\pi+3\pi=21\pi=66\text{ cm}^2$$

Question 3:

A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm (see the given figure). $\left[\text{Use } \pi=\frac{22}{7}\right]$

Solution 3:



It can be observed that

Radius (r) of cylindrical part = Radius (r) of hemispherical part

$$=\frac{2.8}{2}=1.4\text{ cm}$$

Length of each hemispherical part = Radius of hemispherical part = 1.4 cm

Length (h) of cylindrical part = 5 − 2 × Length of hemispherical part

= 5 − 2 × 1.4 = 2.2 cm

Volume of one gulab jamun = Vol. of cylindrical part

+ 2 × Vol. of hemispherical part

$$= \pi r^2 h + 2 \times \frac{2}{3}\pi r^3 = \pi r^2 h + \frac{4}{3}\pi r^3$$

$$= \pi \times (1.4)^2 \times (2.2) + \frac{4}{3}\pi (1.4)^3$$

$$= \frac{22}{7} \times 1.4 \times 1.4 \times 2.2 + \frac{4}{3} \times \frac{22}{7} \times 1.4 \times 1.4 \times 1.4$$

$$= 13.552 + 11.498 = 25.05 \text{ cm}^3$$

Volume of 45 gulab jamuns = 45 × 25.05 = 1,127.25 cm^3

Volume of sugar syrup = 30% of volume

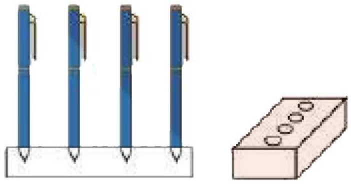
$$= \frac{30}{100} \times 1{,}127.25$$

$$= 338.17 \text{ cm}^3$$

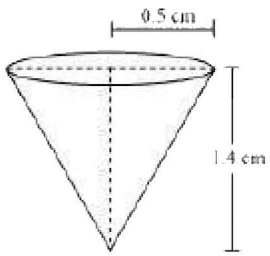
$$\cong 338 \text{ cm}^3$$

Question 4:

A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboids are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand (see the following figure). $\left[\text{Use } \pi = \frac{22}{7}\right]$



Solution 4:



Depth (h) of each conical depression = 1.4 cm
Radius (r) of each conical depression = 0.5 cm
Volume of wood = Volume of cuboid − 4 × Volume of cones

$$= l \times b \times h - 4 \times \frac{1}{3}\pi r^2 h$$

$$= 15 \times 10 \times 3.5 - 4 \times \frac{1}{3} \times \frac{22}{7} \times \left(\frac{1}{2}\right)^2 \times 1.4$$

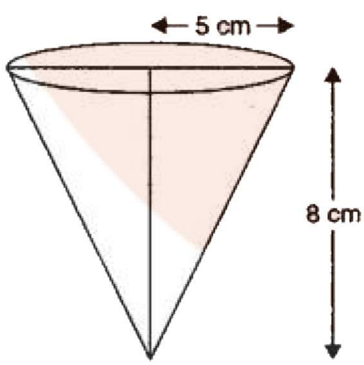
$$= 525 - 1.47$$

$$= 523.53 \text{ cm}^3$$

Question 5:

A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Solution 5:



Height (h) of conical vessel = 8 cm
Radius (r_1) of conical vessel = 5 cm
Radius (r_2) of lead shots = 0.5 cm

Let *n* number of lead shots were dropped in the vessel.

Volume of water spilled = Volume of dropped lead shots

$$\frac{1}{4} \times \text{volume of cone} = n \times \frac{4}{3} r_2^{\ 3}$$

$$\frac{1}{4} \times \frac{1}{3} \pi r_1^{\ 2} h = n \times \frac{4}{3} \pi r_2^{\ 3}$$

$$r_1^{\ 2} h = n \times 16 r_2^{\ 3}$$

$$5^2 \times 8 = n \times 16 \times (0.5)^3$$

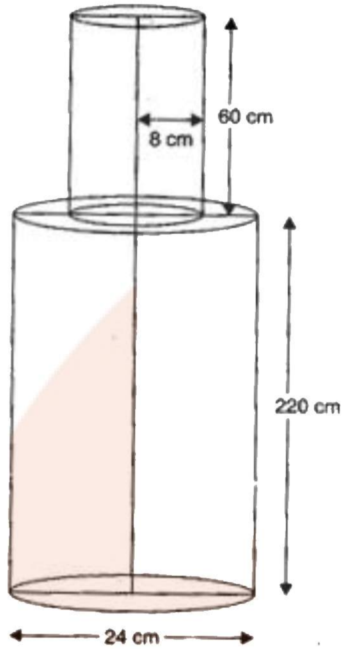
$$n = \frac{25 \times 8}{16 \times \left(\frac{1}{2}\right)^3} = 100$$

Hence, the number of lead shots dropped in the vessel is 100.

Question 6:

A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 g mass. [Use π = 3.14]

Solution 6:



From the figure, it can be observed that
Height (h_1) of larger cylinder = 220 cm

Radius (r_1) of larger cylinder = $\frac{24}{2}$ = 12 cm

Height (h_2) of smaller cylinder = 60 cm

Radius (r_2) of smaller cylinder = 8 cm

Total volume of pole = Volume of larger cylinder + volume of smaller cylinder

$= \pi r_1^2 h_1 + \pi r_2^2 h_2$

$= \pi (12)^2 \times 220 + \pi (8)^2 \times 60$

$= \pi [144 \times 220 + 64 \times 60]$

$= 35520 \times 3.14 = 1,11,532.8 \text{ cm}^3$

Mass of 1cm^3 iron = 8 g

Mass of 111532.8cm^3 iron = 111532.8 × 8 = 892262.4 g = 892.262 kg

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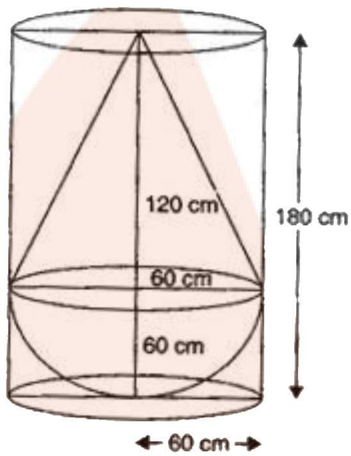
Question 7:

A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Solution 7:



Radius (r) of hemispherical part = Radius (r) of conical part = 60 cm

Height (h_2) of conical part of solid = 120 cm

Height (h_1) of cylinder = 180 cm

Radius (r) of cylinder = 60 cm

Volume of water left = Volume of cylinder − Volume of solid

= Volume of cylinder – (Volume of cone + Volume of hemisphere)

$$= \pi r^2 h_1 - \left(\frac{1}{3}\pi r^2 h_2 + \frac{2}{3}\pi r^3\right)$$

$$= \pi(60)^2(180) - \left(\frac{1}{3}\pi(60)^2 \times 120 + \frac{2}{3}\pi(60)^3\right)$$

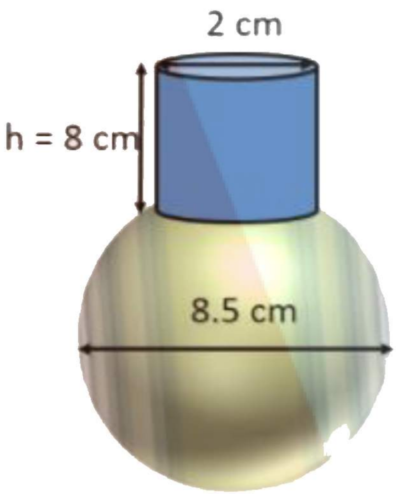
$$= \pi(60)^2\left[(180) - (40 + 40)\right]$$

$$= \pi(3600)(100) = 3,60,000\pi \text{ cm}^3 = 11311428.57 \text{ cm}^3 = 1.131 \text{ m}^3$$

Question 8:

A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3. Check whether she is correct, taking the above as the inside measurements, and $\pi = 3.14$.

Solution 8:



Height (h) of cylindrical part = 8 cm

Radius (r_2) of cylindrical part = $\frac{2}{2} = 1\text{cm}$

Radius (r_1) spherical part $= \frac{8.5}{2} = 4.25\text{cm}$

Volume of vessel = Volume of sphere + Volume of cylinder

$$= \frac{4}{3}\pi r_1^3 + \pi r_2^2 h$$

$$= \frac{4}{3}\pi\left(\frac{8.5}{2}\right)^3 + \pi(1)^2(8)$$

$$= \frac{4}{3} \times 3.14 \times 76.765625 + 8 \times 3.14$$

$$= 321.392 + 25.12$$

$$= 346.512$$

$$= 346.51\text{ cm}^3$$

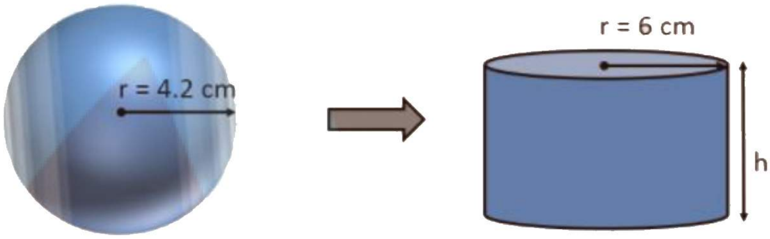
Hence, she is wrong

EXERCISE NO:13.3

Question 1:

A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

Solution 1:



Radius ($r1$) of hemisphere = 4.2 cm

Radius (r_2) of cylinder = 6 cm

Let the height of the cylinder be h.

The object formed by recasting the hemisphere will be the same in volume.

Volume of sphere = Volume of cylinder

$$\frac{4}{3}\pi r_1^3 = \pi r_2^2 h$$

$$\frac{4}{3}\pi(4.2)^3 = \pi(6)^2 h$$

$$\frac{4}{3}\times\frac{4.2\times4.2\times4.2}{36} = h$$

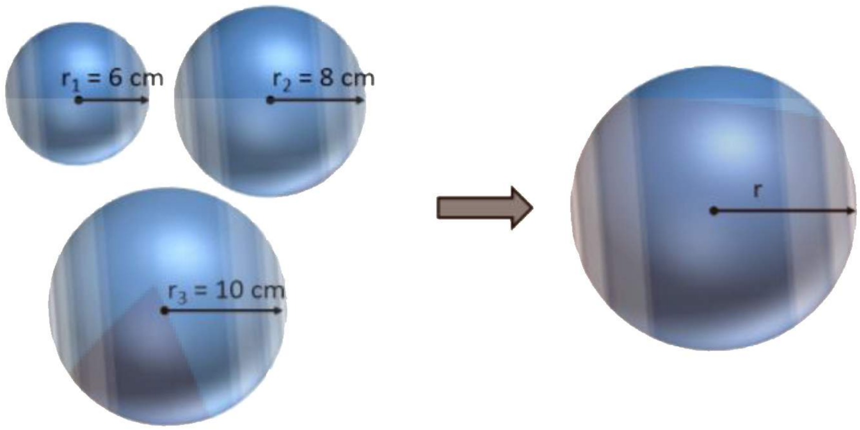
$$h = (1.4)^3 = 2.74 \text{ cm}$$

Hence, the height of the cylinder so formed will be 2.74 cm.

Question 2:

Metallic spheres of radii 6 cm, 8 cm, and 10 cm, respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

Solution 2:



Radius (r_1) of 1st sphere = 6 cm

Radius (r_2) of 2nd sphere = 8 cm

Radius (r_3) of 3rd sphere = 10 cm

Let the radius of the resulting sphere be r.

The object formed by recasting these spheres will be same in volume as the sum of the volumes of these spheres.

Volume of 3 spheres = Volume of resulting sphere

$$\frac{4}{3}\pi\left[r_1^3 + r_2^3 + r_3^3\right] = \frac{4}{3}\pi r^3$$

$$\frac{4}{3}\pi\left[6^3 + 8^3 + 10^3\right] = \frac{4}{3}\pi r^3$$

$$r^3 = 216 + 512 + 1000 = 1728$$

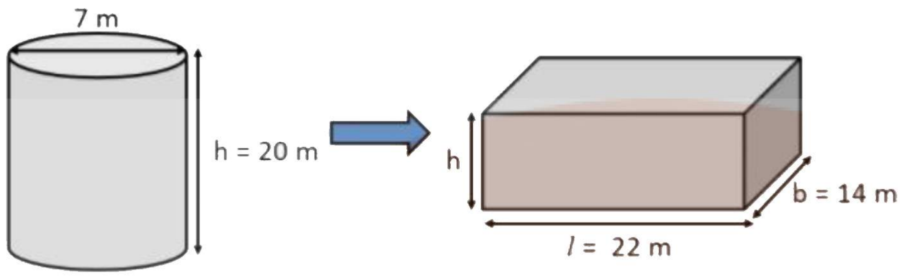
$$r = 12 \text{ cm}$$

Therefore, the radius of the sphere so formed will be 12 cm.

Question 3:

A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Solution 3:



The shape of the well will be cylindrical.

Depth (h) of well = 20 m

Radius (r) of circular end of well = $\frac{7}{2}$ m

Area of platform = Length × Breadth = 22 × 14 m^2

Let height of the platform = H

Volume of soil dug from the well will be equal to the volume of soil scattered on the platform.

Volume of soil from well = Volume of soil used to make such platform

$\pi \times r^2 \times h$ = Area of platform × Height of platform

$$\pi \times \left(\frac{7}{2}\right)^2 \times 20 = 22 \times 14 \times H$$

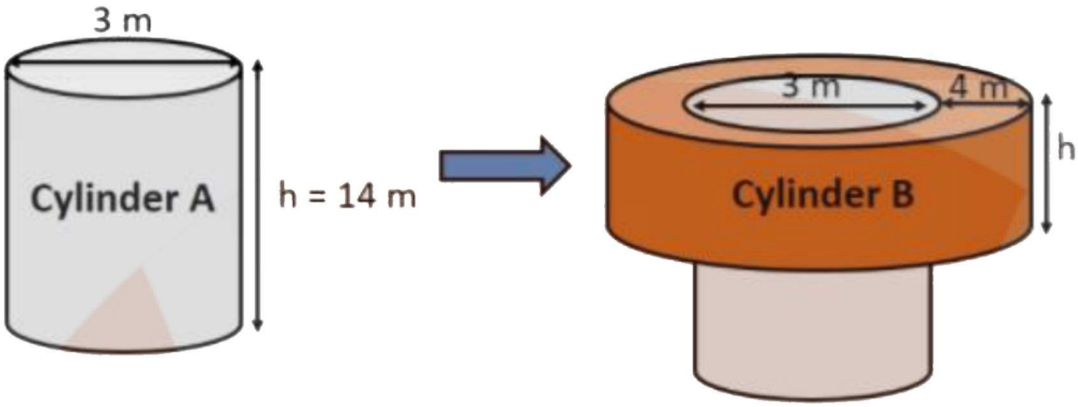
$$\therefore\ H = \frac{22}{7} \times \frac{49}{4} \times \frac{20}{22 \times 14} = \frac{5}{2} m = 2.5m$$

Therefore, the height of such platform will be 2.5 m.

Question 4:

A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment.

Solution 4:



The shape of the well will be cylindrical.

Depth (h_1) of well = 14 m

Radius (r_1) of the circular end of well = $\frac{3}{2}$ m

Width of embankment = 4 m

From the figure, it can be observed that our embankment will be in a cylindrical

shape having outer radius (r_2) as $4+\frac{3}{2}=\frac{11}{2}$ m and inner radius (r_1) as $\frac{3}{2}$ m.

Let the height of embankment be h_2.

Volume of soil dug from well = Volume of earth used to form embankment

$$\pi \times r_1^2 \times h_1 = \pi \times \left(r_2^2 - r_1^2\right) \times h_2$$

$$\pi \times \left(\frac{3}{2}\right)^2 \times 14 = \pi \times \left[\left(\frac{11}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right] \times h$$

$$\frac{9}{4} \times 14 = \frac{112}{4} \times h$$

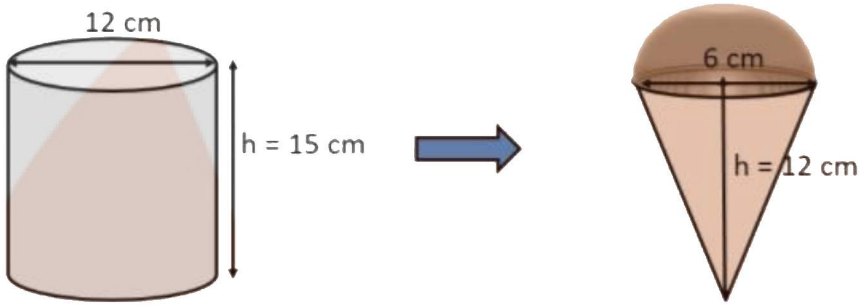
$$h = \frac{9}{8} = 1.125 \text{ m}$$

Therefore, the height of the embankment will be 1.125 m.

Question 5:

A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.

Solution 5:



Height (h_1) of cylindrical container = 15 cm

Radius (r_1) of circular end of container = $\frac{12}{2} = 6\text{cm}$

Radius (r_2) of circular end of ice-cream cone = $\frac{6}{2} = 3\text{cm}$

Height (h_2) of conical part of ice-cream cone = 12 cm

Let n ice-cream cones be filled with ice-cream of the container.

Volume of ice-cream in cylinder = n × (Volume of 1 ice-cream cone + Volume of hemispherical shape on the top)

$$\pi r_1^2 h_1 = n\left(\frac{1}{3}\pi r_2^2 h_2 + \frac{2}{3}\pi r_2^3\right)$$

$$n = \frac{6^2 \times 15}{\frac{1}{3}\times 9 \times 12 + \frac{2}{3}\times (3)^2}$$

$$n = \frac{36 \times 15 \times 3}{108 \times 54}$$

$$n = 10$$

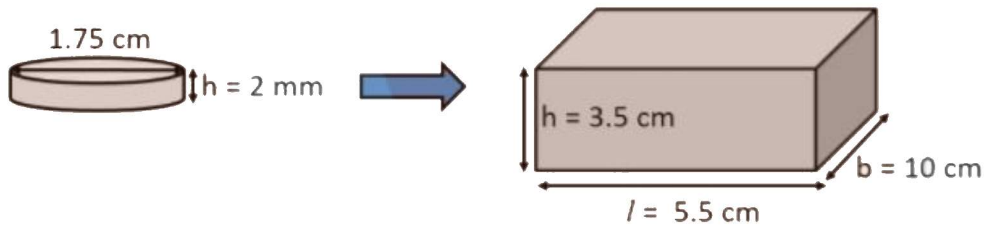
Therefore, 10 ice-cream cones can be filled with the ice-cream in the container.

Question 6:

How many silver coins, 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimensions $5.5\ \text{cm} \times 10\text{cm} \times 3.5\ \text{cm}$?

$\left[\text{Use } \pi = \frac{22}{7}\right]$

Solution 6:



Coins are cylindrical in shape.

Height (h_1) of cylindrical coins = 2 mm = 0.2 cm

Radius (r) of circular end of coins $= \frac{1.75}{2} = 0.875$ cm

Let n coins be melted to form the required cuboids.

Volume of n coins = Volume of cuboids

$$n \times \pi \times r^2 \times h_1 = l \times b \times h$$

$$n \times \pi \times (0.875)^2 \times 0.2 = 5.5 \times 10 \times 3.5$$

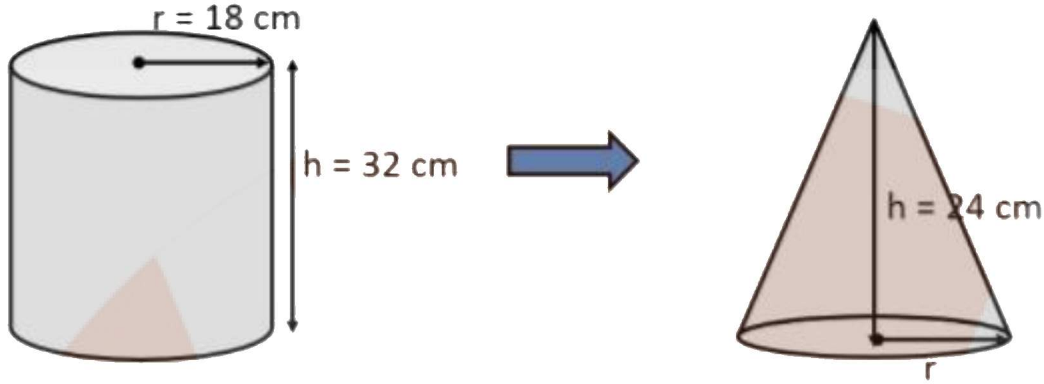
$$n = \frac{5.5 \times 10 \times 3.5 \times 7}{(0.875)^2 \times 0.2 \times 22} = 400$$

Therefore, the number of coins melted to form such a cuboid is 400.

Question 7:

A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm. Find the radius and slant height of the heap.

Solution 7:



Height (h_1) of cylindrical bucket = 32 cm

Radius (r_1) of circular end of bucket = 18 cm

Height (h_2) of conical heap = 24 cm

Let the radius of the circular end of conical heap be r_2.

The volume of sand in the cylindrical bucket will be equal to the volume of sand in the conical heap.

Volume of sand in the cylindrical bucket = Volume of sand in conical heap

$$\pi \times r_1^2 \times h_1 = \frac{1}{3}\pi \times r_2^2 \times h_2$$

$$\pi \times 18^2 \times 32 = \frac{1}{3}\pi \times r_2^2 \times 24$$

$$\pi \times 18^2 \times 32 = \frac{1}{3}\pi \times r_2^2 \times 24$$

$$r_2^2 = \frac{3 \times 18^2 \times 32}{24} = 18^2 \times 4$$

$$r_2 = 18 \times 2 = 36 \text{ cm}$$

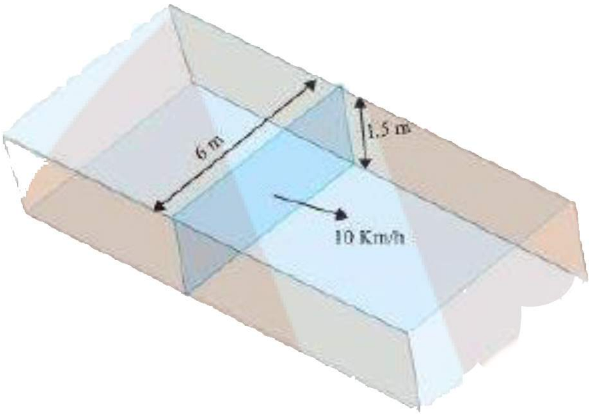
Slant height $= \sqrt{36^2 + 24^2} = \sqrt{12^2 \times \left(3^2 + 2^2\right)} = 12\sqrt{13}$ cm

Therefore, the radius and slant height of the conical heap are 36 cm and $12\sqrt{13}$ cm respectively.

Question 8:

Water in canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. how much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?

Solution 8:



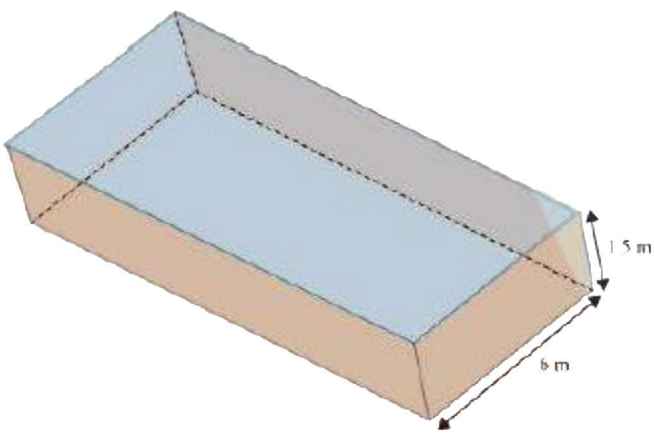
Consider an area of cross-section of canal as ABCD.

Area of cross-section = $6 \times 1.5 = 9\ m^2$

Speed of water = 10 km/h= $\frac{10000}{60} metre/min$

Volume of water that flows in 1 minute from canal $= 9 \times \frac{10000}{60} = 1500\ m^3$

Volume of water that flows in 30 minutes from canal = $30 \times 1500 = 45000 m^3$



Let the irrigated area be A. Volume of water irrigating the required area will be equal to the volume of water that flowed in 30 minutes from the canal

Vol. of water flowing in 30 minutes from canal = Vol. of water irrigating the reqd. area

$$45000 = \frac{A \times 8}{100}$$

A = 562500 m^2

Therefore, area irrigated in 30 minutes is 562500 m^2.

Question 9:

A farmer connects a pipe of internal diameter 20 cm form a canal into a cylindrical tank in her field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled?

Solution 9:

Consider an area of cross-section of pipe as shown in the figure.

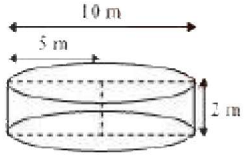
Radius (r_1) of circular end of pipe = $\frac{20}{200} = 0.1$ m

Area of cross-section = $\pi \times r_1^2 = \pi \times (0.1)^2 = 0.01\ \pi m^2$

Speed of water = 3 km/h = $\frac{3000}{60} = 50$ meter/min

Volume of water that flows in 1 minute from pipe = 50 × 0.01π = 0.5π m^3

Volume of water that flows in t minutes from pipe = t × 0.5π m^3



Radius (r_2) of circular end of cylindrical tank = $\frac{10}{2} = 5$m

Depth (h_2) of cylindrical tank = 2 m

Let the tank be filled completely in t minutes.

Volume of water filled in tank in t minutes is equal to the volume of water flowed in t minutes from the pipe.

Volume of water that flows in t minutes from pipe = Volume of water in tank

$t \times 0.5\pi = \pi \times (r_2)^2 \times h_2$
$t \times 0.5 = (5)^2 \times 2$
$t = 100$
Therefore, the cylindrical tank will be filled in 100 minutes.

ASSIGNMENT
SURFACE AREAS AND VOLUMES

1. A tent is of the shape of a right circular cylinder upto a height of 3 metres and then becomes a right circular cone with a maximum height of 13.5 m above the ground. Calculate cost of painting the inner side of the tent at the rate of Rs. 2 per square metre, if radius of base is 14 m.

2. A circus tent is cylindrical upto a height of 3 m and conical above it. If the diameter of the base is 105 m and the slant height of the conical part is 53 m, find the total canvas used in making the tent.

3. A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 19 cm and the diameter of the cylinder is 7 cm. Find volume and total surface area of the solid.

4. A solid toy is in the form of a right circular cylinder with a hemispherical shape at one end and a cone at the other end. Their common diameter is 4.2 cm and the height of the cylindrical and conical portions are 12 cm and 7 cm respectively. Find the volume of the solid toy.

5. A solid is composed of a cylinder with hemispherical ends. It the whole length of the solid is 104 cm and the radius of each of the hemispherical ends is 7 cm, find the cost of polishing its surface at the rate of Rs. 10 per dm^2.

6. A cylindrical tub of radius 5 cm and length 9.8 cm is full of water. A solid in the form of a right circular cone mounted on a hemisphere is 3.5 cm and height of the cone outside the hemisphere is 5 cm, find the volume of the water left in the tub.

7. A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The radius and height of the cylindrical part are 5 cm and 13 cm respectively. The radii of the hemispherical and conical parts are the same as that of the cylindrical parts. Find the surface area of the toy if the total height of the toy is 30 cm.

8. A cylindrical container of radius 6 cm and height 15 cm is filled with ice- cream. The whole ice-cream has to be distributed to 10 children in equal cones with hemispherical tops. If the height of the conical portion is four times the radius of its base, find the radius of the ice-cream cone.

9. A building is in the form of a cylinder surmounted by a hemispherical vaulted dome and contains $41\frac{19}{21}$ cm^3 of air. If the internal diameter of the building is equal to the total height above the floor, find the height of the building.

10. An iron pillar has some part in the form of a right circular cylinder and the remaining in the form of a right circular cone. The radius of the base of each of the cone and the cylinder is 8 cm, the cylindrical part is 240 cm high and conical part is 36 cm high. Find the weight of the pillar, if 1 cm^3. of iron weighs 10 grams.

ANSWERS

1. Rs 2068 **2.** 9735 m^2 **3.** 641.66 cm^3, 418 cm^2 **4.** 218.064 cm^3 **5.** Rs. 457.60

6. 616 cm^3 **7.** 770 cm^2 **8.** 3 cm **9.** 4 m **10.** 506.88 kg